

**Mathematics Standards**

**Calculus**

**Course Overview:** This course will include the study of change functions, the relationship between distance, velocity, and acceleration, Riemann sums, limits, continuity, instantaneous rates of change, derivatives, curve analysis, advanced derivative tools, applications of derivatives, area functions, area between two curves, integration tools, and applications of integration.

**Bold standards are essential standards that all students will learn as they complete the course.**

**Unit 1 – A Beginning Look at Calculus (approximately 30 days)**

**Description:** In this unit, students will review topics from previous courses such as piecewise functions, compositions, inverses, even & odd functions, domain & range, and horizontal and vertical asymptotes. They will develop the concepts of slope and slope functions and study how particular functions change by examining finite differences. They will also examine both the velocity and distance graph of an object in motion to find average velocity and acceleration.

**Standards**

1. Students will investigate the relationship between distance and velocity.

2. Students will graph piecewise functions, paying particular attention to domain.

3. Students will determine if graphs are continuous or have holes and/or vertical asymptotes.

4. Students will investigate end behavior and how it relates to horizontal asymptotes.

5. **Students will learn to write approach statements.**

6. Students will review composite functions and inverse functions.

7. Students will draw conclusions about even and odd functions based on given information.

8. Students will examine the finite differences of constant, linear, quadratic and cubic functions and find patterns.

9. Students will use finite differences to write slope statements about non-polynomial parent graphs.

10. Students will use a motion detector to “walk” a graph.

11. Students will connect physical motion with position graphs – moving forward/backward and speeding up/slowing down.

12. **Students will explore what a position graph can reveal about velocity and what a velocity graph can reveal about position.**

13. Students will distinguish between the concepts of displacement, total distance traveled, and average velocity.

14. Students will discover how to find an average rate of change on a rate graph, such as finding average velocity on a position graph.

15. Students will learn about acceleration from a velocity graph.

16. Students will investigate situations describing positive or negative velocity with positive or negative acceleration.

17. Students will find velocity from a position graph and find position from a velocity graph.

**Unit 2 – Rates, Sums, Limits, and Continuity** **(approximately 25 days)**

**Description:** In this unit, students will be introduced to all three of the core concepts of calculus: limits, rates of change, and area under a curve. They will approximate the area under a curve using Riemann sums and predict function behavior with limits. The approach statements from the last chapter will lead the concept of limits. They will use limits to define continuity which provides the basis for the Intermediate Value Theorem and they will develop a method to approximate instantaneous rate of change.

**Standards**

1. **Students will approximate area under a curve using rectangles and trapezoids.**

2. Students will approximate area under a curve using Riemann sums.

3. Students will investigate the informal definition of limits as predictions.

4. Students will understand what it means for a limit to exist, informally.

5. **Students will use limits to determine continuity.**

6. Students will us the three-part definition of continuity to investigate why a particular graph is or is not continuous at a particular *x*-value.

7. Students will use their knowledge of continuity to understand of the Intermediate Value Theorem.

8. **Students will calculate limits algebraically.**

9. Students will use limit statements to describe the behavior of a function.

10. **Students will use slopes of secant lines to estimate velocity at a point.**

11. Students will investigate some special limits that involve trig functions.

12. Students will examine under- and over-approximations on increasing and decreasing functions.

13. Students will estimate the area under a curve using a large number of rectangles.

**Unit 3 – Slope and Curve Analysis** **(approximately 25 days)**

**Description:** In this unit, students will discover the Power Rule of differentiation by finding patterns among the slopes of tangent lines. They will then use limits, secants and tangents to formalize a definition of instantaneous rates of change and this will lead to the formal definition of the derivative. They will use the definition of the derivative to derive the derivatives of many familiar parent functions. They will discover the role of the first and second derivatives to describe a function’s shape, including where it is increasing or decreasing and its concavity. They will also investigate conditions under which a function is and is not differentiable at a point and begin to find antiderivatives.

**Standards**

1. **Students will investigate the slope of polynomial Parent Graphs and infer the Power Rule.**

2. Students will use the slope of a secant line to find the average rate of change.

3. Students will use the slope of a tangent line to find the instantaneous rate of change.

4. **Students will use limits to establish the definition of the derivative.**

5. **Students will choose an appropriate method to find a slope function.**

6. Students will discover the derivative of sine and cosine graphically.

7. Students will understand the graphical significance of the derivative.

8. Students will examine the effect that increasing and decreasing slopes have on the shape of the curve and develop vocabulary about concavity.

9. Students will sketch first and second derivatives of a given function.

10. Students will connect concavity with the sign of the second derivative.

11. Students will explore situations where the derivative does not exist at a particular point.

12. **Students will find an antiderivative of a polynomial.**

13. Students will analyze some unusual functions and verify that the graphing calculator can occasionally give misleading results.

**Unit 4 – The Fundamental Theorem of Calculus (approximately 25 days)**

**Description:** In this unit, students will be introduced to integrals as the limit of Riemann sums. They will learn to evaluate these limits on your calculator and geometrically (when possible) and they will develop a list of the properties of integrals. They will use algebra and geometry to discover area functions and they will recognize the relationship between derivatives and integrals leading to the Fundamental Theorem of Calculus. They will also be using multiple strategies to find the area between curves, deciding which strategy is best for each situation.

**Standards**

1. Students will use a Riemann sum to write an expression representing the exact area under a function.

2. **Students will express area as a definite integral.**

3. Students will use the numerical integration feature of their graphing calculators to calculate the area under a curve of a definite integral.

4. Students will discover and apply the basic properties of integrals with general limits of integration and a general function.

5. Students will explore limits of integration where only one of the limits is a constant.

6. Students will find general area functions.

7. **Students will distinguish between definite and indefinite integrals.**

8. Students will use area functions to numerically integrate.

9. **Students will evaluate definite integrals using the Fundamental Theorem of Calculus.**

10. Students will define the relationship between an integral and its derivative.

11. Students will explain both parts of the fundamental Theorem of Calculus.

12. **Students will use their knowledge of velocity, position, acceleration, and the Fundamental Theorem of Calculus to solve real-world application problems.**

13. Students will set up integrals for finding the area between two curves.

14. Students will find the area between two curves using a variety of methods.

**Unit 5 – Optimization and Derivative Tools** **(approximately 25 days)**

**Description:** In this unit, students will use the derivative in a variety of applied situations including velocity, acceleration, and optimization. They will learn techniques to find the derivative of more complicated functions involving products, quotients, and composite functions. They will apply the ideas of first and second derivatives to solve optimization problems and they will evaluate limits using l’Hôpital’s Rule.

**Standards**

1. **Students will find velocity and acceleration from a position function.**

2. Students will find the dimensions of various box designs that will maximize the volume.

3. Students will describe the shape of a curve given information about the first and second derivative for a particular interval and a particular point.

4. Students will apply the first and second derivative tests.

5. **Students will learn and apply the Product Rule for derivatives.**

6. Student will explore composite functions and their derivatives.

7. **Students will find a process for taking the derivative of composite functions (the Chain Rule).**

8. Students will apply the Chain Rule to find derivatives of composite functions.

9. **Students will learn and apply the Quotient Rule for derivatives.**

10. Students will find the derivatives of the tangent, cotangent, secant, and cosecant functions from the derivatives of sine and cosine using the Quotient Rule and/or Chain Rule.

11. Students will solve a variety of optimization problems.

12. Students will explore limits of indeterminate forms.

13. Students will use l’Hôpital’s rule to find limits of indeterminate forms.

**Unit 6 – More Derivative Tools** **(approximately 15 days)**

**Description:** In this unit, students will learn how take derivatives and integrals of exponential functions and they will learn how to use implicit differentiation to find derivatives of relations defining implicit functions. They will use implicit differentiation to develop a technique to find the derivative of inverse functions and they will use these tools to find the derivatives of logarithmic and inverse trigonometric functions.

**Standards**

1. Students will formulate an understanding for the value of *e* (Euler’s number).

2. **Students will gain an intuitive understanding of the derivative of *ex*.**

3. Students will explore the derivatives of ex and other exponential functions.

4. Students will use the Power, Product, Quotient, and Chain Rules on logarithmic and trigonometric functions.

5. Students will learn to integrate exponential functions.

6. Students will take the derivatives of trigonometric and exponential composites of three functions using the Power, Chain, Product, and/or Quotient Rules.

7. Students will use implicit differentiation to find derivatives of relations defining implicit functions.

8. Students will find the derivatives of the inverse trigonometric functions through implicit differentiation and substitution, with the aid of a reference triangle in a unit circle.

9. Students will learn the derivatives of logarithmic functions.

**Unit 7 – Related Rates** **(approximately 10 days)**

**Description:** In this unit, students will write related rate statements and understand how two different rates can be mutually dependent. They will progress to solving problems involving related rates.

**Standards**

1. Students will describe the rates of change of objects that are directly related.

2. Students will solve an introductory real-world related rate problem.

3. Students will use proportional triangles to solve related rate problems.

4. Students will solve related rates problems involving geometry and the distance formula.

5. Students will solve related rates problems involving trigonometry.

**Unit 8 – Volumes of Revolution** **(approximately 15 days)**

**Description:** In this unit, students will find the volumes of solids of revolution. They will use the method of slices (disks) or washers and the method of cylindrical shells.

**Standards**

1. **Students will explore finding the volume of objects by slicing.**

2. Students will develop a spatial understanding to discover how to slice.

3. Students will draw a graph of a curve revolved around the *x*-axis showing the typical slice or disk.

4. Students will set up an integral accumulating the volumes of the disks.

5. Students will decide when to slice the solid vertically or horizontally.

6. Students will compute the volume of solids of revolution using disks and washers.

7. Students will use integrals to find the volume for regions rotated about lines other than the *x*- and *y*-axes.

8. Students will write integrals representing the volume of solids using cylindrical shells.

9. Students will decide on the appropriate method (slices or shells) for finding volumes for a variety of solids of revolution.